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# PRECISE MEASURING WITH INVAR WIRES

AND

# THE MEASUREMENT OF KOOTENAY BASE

(According to the method of the International
Bureau of Weights and Measures)



BY

P. A. CARSON, D.L.S.,

Under the direction of The Surveyor General of Canada

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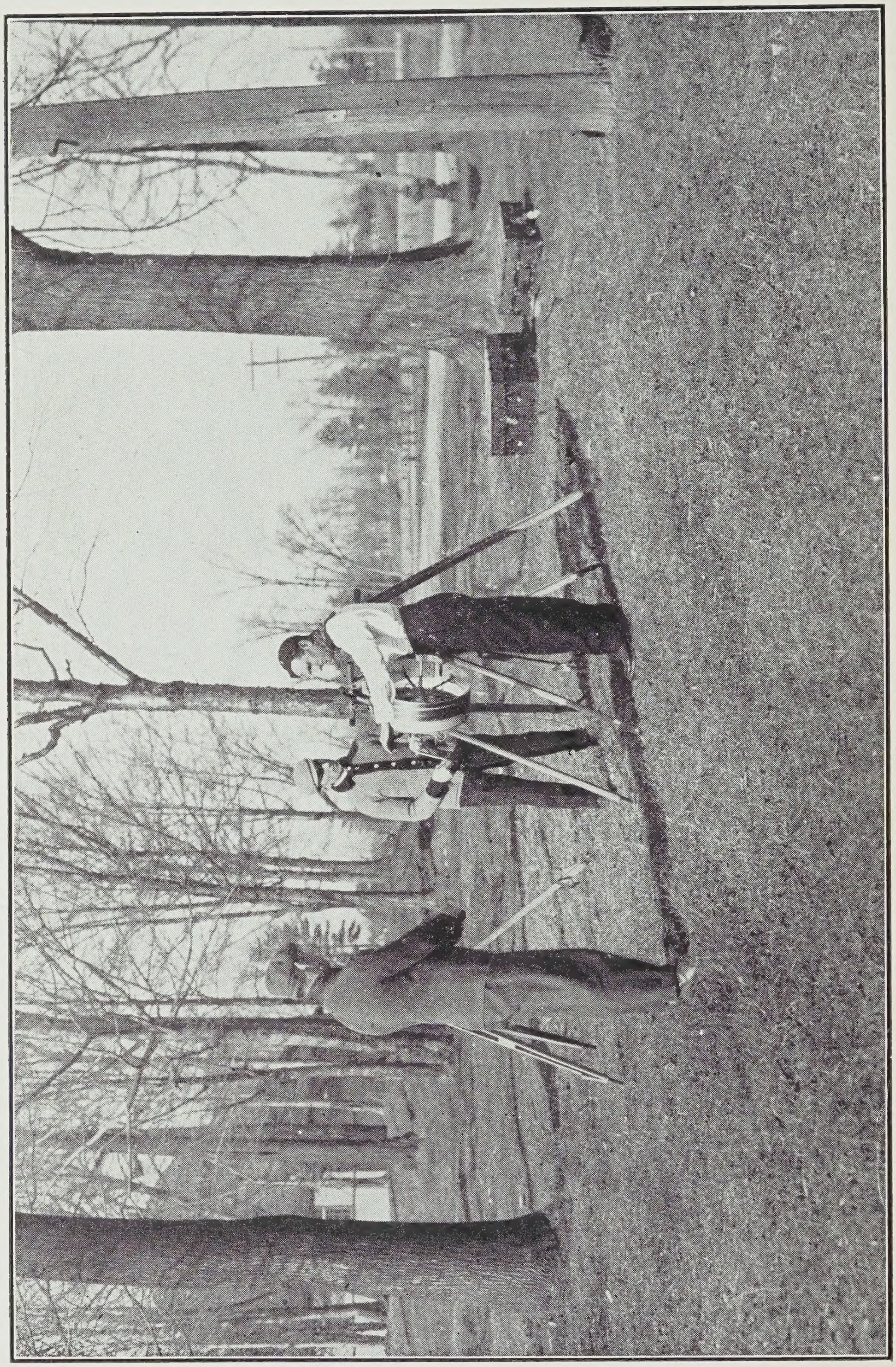
PLATE I.



UNROLLING THE INVAR WIRES

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# PRECISE MEASURING WITH INVAR WIRES; AND THE MEASUREMENT OF KOOTENAY BASE.

By P. A. Carson.

THE first essential in measuring linear magnitudes is of course a unit of length. And what a babel of units there is, every age and every people contributing their share to the confusion. Fortunately, however, by the concurrent action of the principal governments of the world, we now have a recognized standard unit, viz.: the international metre, which is copied after the French metre. The international metre is defined as the distance between two lines at $0^\circ$ C on a platinum-iridium bar deposited at the International Bureau of Weights and Measures, Sèvres, France. Prototypes of this standard metre have been formed and distributed amongst the different countries. These are called national prototype standards.

It is obvious, however, that these standards must be guarded with the greatest care, and cannot be used in performing actual measurements. Consequently, the next essential is a measuring unit. This measuring unit generally takes the form of a bar or tape; and the errors in the value of a base measured with such arise from two principal sources, viz.: comparisons and measurements. Errors in the comparisons which determine the length of the measuring bar or tape are accumulative, and it is of the utmost importance that the standardizing of the measuring unit should be performed with the greatest precision. Also that the bar or tape should either have a permanent length or change only according to known laws.

The greatest source of error due to measurements has always been the uncertainty in the temperature, and hence of the length of the measuring unit during the field operations; first, because during changing temperatures the recording thermometers rarely measure the true temperature of the bar or tape, and secondly,

because the volumetric change in metals generally lags behind the temperature change. The earlier measurements of base-lines in connection with geodetic triangulations were performed with wooden contact rods. These were superseded by iron rods, glass tubes, steel bars and steel chains, but in all the errors due to temperature were beyond control. To evade the temperature difficulty many ingenious compensating apparatus were devised, such as the Borda and the Colby apparatus, which were based on the different rates of expansion of two metals. These instruments while theoretically perfect were soon found faulty in practice and were abandoned. The U. S. Coast and Geodetic Survey Iced-bar Apparatus is one of the most precise instruments for base measurement, the temperature difficulty being avoided by packing a steel bar in ice. But this apparatus is very cumbersome and takes a small army of men to handle it. It is, therefore, too expensive for general use. The modern steel tape affords a very exact, rapid and economical method of measuring even primary bases. This method has reached a high degree of perfection in the United States, by taking great precautions for temperature changes and making the measurements at night.

Several years ago M. Ch. Guillaume, Assistant Director of the International Bureau of Weights and Measures, in making researches with alloys of nickel and steel, discovered that an alloy of these two metals containing about thirty-six per cent. of nickel possesses a very small co-efficient of expansion and its use was immediately suggested for base measurement. To this alloy has been given the name "invar," a word derived from the same root as the word "invariable" and having a similar meaning.

By the use of invar in the form of wires, and an ingenious apparatus devised by M. Guillaume (hereinafter described), we now have at our command a base measuring instrument capable of the same accuracy as the most refined bar apparatus, and at the same time much more convenient and economical. It has also many advantages over steel tapes in that the invar wires may be standardized at any Bureau of Standards instead of in the field, greater precision can be attained, the work may be done

in day time, and errors due to uncertain temperatures are almost entirely eliminated.

For the complete manual of the theory of invar and the description of the base-line apparatus, reference should be made to "La Mesure Rapide des Bases Géodésiques" (4th edition) by J. Rene Beniot and Ch-Ed. Guillaume, of the International Bureau of Weights and Measures. It has been considered advisable, however, to give in connection with this report a brief discussion on the theory and peculiarities of that strange alloy termed "invar," together with a description of the salient features of the apparatus, and its application to the rapid and precise measurement of geodetic base lines.

*Properties and Manufacture of Invar:* Alloys of nickel and steel exhibit many peculiar and interesting properties especially in their action under different thermal conditions. This anomaly of dilatation was observed by M. Guillaume. As the percentage of nickel is increased above 25 per cent. the dilatation decreases very rapidly and reaches a minimum with 36 per cent. nickel; then it rises again rapidly, and as the percentage of nickel is increased approaches slowly to the dilatation of pure nickel.

Invar is more like nickel than steel in appearance. It is less easily oxidised than steel, yet requires some care to keep it free from rust, especially in moist climates. This may be done by rubbing with a rag covered with vaseline, as the small spots of rust which sometimes appear on the surface do not adhere very closely, and may be easily removed in this manner. The alloy is very ductile, yet at the same time quite tenacious. It laminates and is made into wires very easily, but quickly wears out files and other hard tools. In the malleable state it will submit to great alterations in shape without rupture, while on the contrary, when it has been laminated or drawn into wires, it reaches a state of elasticity suitable for ordinary springs. The modulus of elasticity of invar in the form of cold-drawn and hammered wires is about 16,000 Kg.: mm.$^2$, its tensile strength being about one-half of that of steel. One of the peculiar properties of invar is that its modulus of elasticity increases with an

increase of temperature which is contrary to all other known metals.

Invar intended for geodetic purposes is made by certain steel manufactories in France, the usual additions of manganese, silicate, and carbon being reduced to a minimum. Specimens of each casting are then examined, and only those which possess the requisite small coefficient of expansion are accepted. These are then made into wires, being forged at very high temperatures, and cold-drawn and hammered at low temperatures. They are reduced to an average diameter of two millimetres by means of steel wire-machines, and then made to the definite diameter of 1·65 mm. through ruby holes, which give a smooth surface and a uniform diameter.

*Étuvage.*—Invar wires so manufactured are, however, by no means ready for use, but must be carefully studied and examined in the laboratory. It has been observed that invar in the course of time shows a gradual permanent lengthening extending over a number of years, which lengthening approaches slowly towards a limit which seems definite. The value of this limit depends on the temperature and the progress is more rapid as the temperature is higher. This permanent lengthening is considerably minimized by a peculiar treatment of the alloy, termed 'étuvage.' The wires are rolled on a spool-like cylindrical boiler of 50 cm. diameter, forming a kind of drum. The boiler is filled with water, which is kept at boiling point (100° C) for several days. This temperature is then lowered gradually so as to reach 40° C or even 25° C in about three months, the alterations in temperature becoming slower as the temperature decreases. By submitting the wires to this process of 'étuvage,' they undergo the series of changes which would have taken a great number of years to accomplish at ordinary temperatures, and which changes will not now take place again. Even after the étuvage process is completed there is still for some time a gradual though small lengthening. This change is rather rapid at first, then becomes less and less and it is wise to keep the wires for some months before attempting to determine their absolute length.

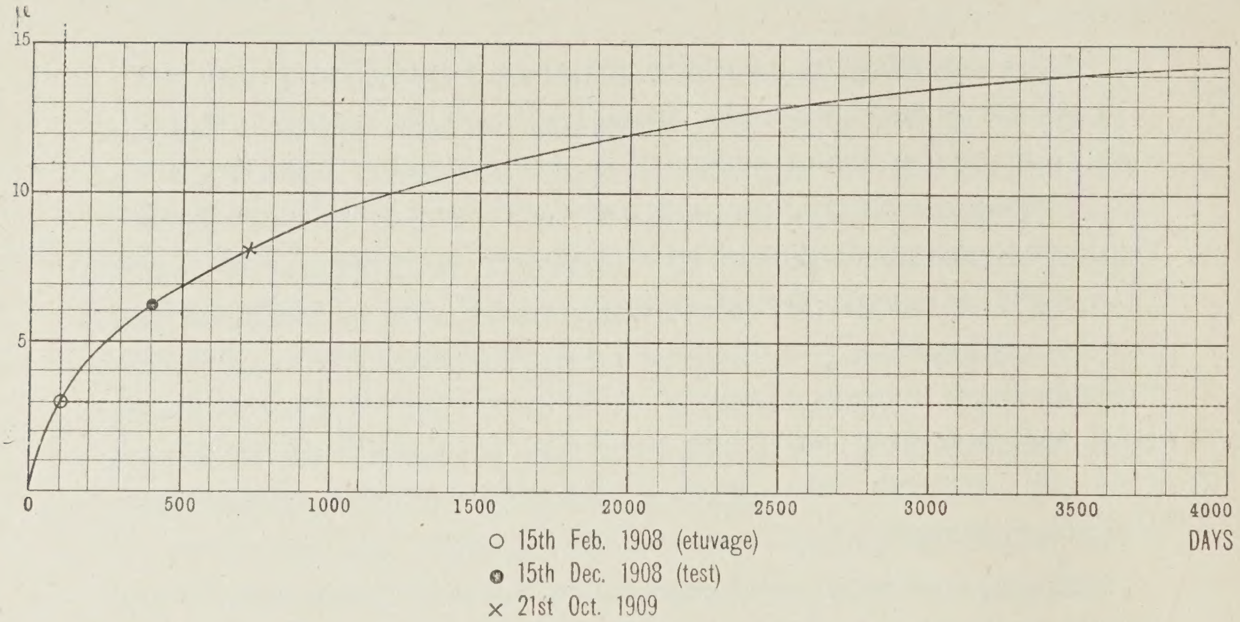



FIG. 1

The curve in Fig. 1 shows the gradual permanent lengthening, which takes place in the course of time, in an invar bar, one metre long, which has been treated by the étuvage operation, the temperature being reduced to 40° C, then kept at that of the laboratory (6° C to 22° C). The abscissæ represent the number of days after étuvage and the ordinates the lengthening in microns ($\mu$) per metre (a micron being the one-millionth part of a metre).

When the temperature of the étuvage is lowered to 25° C instead of 40° C, the origin of the curve is decreased by one hundred days and three microns, as shown by the dotted lines in the figure.

Record is kept of the dates of étuvage, and of the standardizing tests, and from the curve we can determine what alteration in length has taken place in the interim. When the wires are used for field measurements we must note the date and reckon the number of days since étuvage or test. By plotting off the correct distance along the abscissa the curve will show in microns per metre the gradual lengthening of the wire, and

therefore we have the quantity which must be added to the value of the wire obtained in the standardization.

The corrections given by the curve ought to be considered at least for the first two years, only as an approximation to the millionth part.

*Periodic and Daily Temperature Dilatation.*—Although the expansion of invar due to temperature is very small, it is by no means negligible. When the temperature of invar is altered a change in length takes place. This change is of two kinds, termed daily and periodic, the first of which is by far the greater, and occurs simultaneously with the change in temperature. The periodic change, which is very small, follows the other slowly, and corresponds to the lag of other metals. When the new temperature is greater than the first, the periodic variation is a contraction and conversely. In order to apply this second correction for the thermal variation we proceed as follows: — If we have an invar wire to be used in measuring a base line, we consider the mean temperature for the weeks which precede the measurements to be a temperature which is slowly reached, and the periodic variation will tend to approach the change corresponding to this mean temperature. The following formula of periodic dilatation has been established by observations with the comparator:—

$$\Delta l = -0{\cdot}00325 \times 10^{-6} \times l\, t^2.$$

$t$ being the temperature reckoned from zero.

The numerical coefficient of this formula is very small, consequently an error of five or six degrees does not affect the length of the wire one in a million.

To determine the periodic temperature dilatation of invar wires Nos. 272 and 273, which were used in the measurement of Kootenay base, the maximum and minimum temperatures of the wires were recorded each day for the fortnight preceding their use. The mean temperature for this period was 7°·4 C, and is practically the same as the mean temperature of the wires during the measurements.

We, therefore, have from the above formula

$$\Delta\, l = -\ 0{\cdot}00325 \times 10^{-6} \times (7{\cdot}4^2 - 15^2)\ 24^{m} = +\ 0^{mm}{\cdot}0133.$$

This quantity may be added once for all to the value of the wires for the measurements under consideration.

*Daily Temperature Dilatation.*—The ordinary changes in the temperature recorded during the actual measurements determine the daily thermal corrections, which are applied from the regular coefficient of expansion. For wires Nos. 272 and 273 this formula for the range of ordinary temperatures is

$$\Delta\, l = (-\ 0{\cdot}121 + 0{\cdot}00015\ t)\ 10^{-6}\ t\, l$$

and when expanded in a table for each degree from 0° C to say 30° C, gives the correction to be applied to the length of the wire for each recorded temperature.

The above formula is not the formula of absolute expansion but is the dilatation of the wires when under a tension of ten kgs. The elastic deformations decrease as the temperature increases, therefore this expansion of invar is slightly less than the free or absolute dilatation. It should be noted that this formula is negative, as the lengths of the wires decrease as the temperature rises.

The thermal expansion of steel is, therefore, about one hundred times as great as that of invar wires Nos. 272 and 273.

*Mechanical Treatment of Wires to assure permanence in length.*—As geodetic wires must necessarily be rolled for transportation, it is important to note what effect the continual rolling and unrolling might have on the length of the wires. Numerous observations have shown that an invar wire, cold-drawn and hammered, may be rolled in a diameter of fifty c.m. without suffering any alteration in length. Each wire before it leaves the Bureau is submitted to numerous rollings and unrollings for an extended period of time, and observations are made upon its length before and after until permanence is assured. Great care must be exercised that no portion of a wire shall be rolled in a diameter

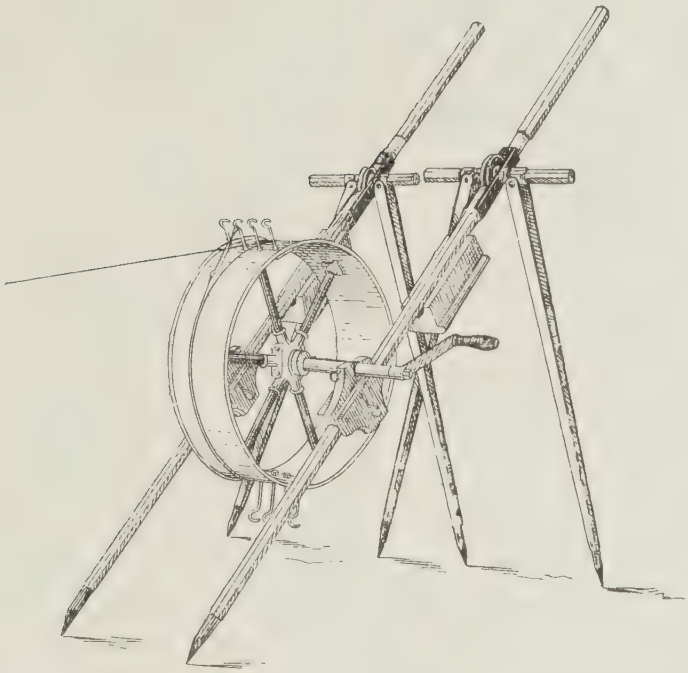


FIG. 2.

of less than fifty cm., lest a degree of instability may be caused which will produce permanent changes of which the wire can rid itself only very slowly.

The wires for transportation are rolled on an aluminum drum of fifty cm. diameter as shown in figure 2. The six spokes of the drum are of steel ; the outer rim is of aluminum, is about sixteen cm. wide, and will hold four twenty-four metre wires. The ends of the wires are fixed to two pairs of hooks fastened to the rim of the drum so that the end scales are tangential to the circumference. For winding and unwinding, the drum is fixed to a steel axle with a handle, and the whole is attached to the two tension tripods placed side by side. When not in use the drum and wires are kept in a specially designed trunk. The wires should be rolled rather loosely on the drum so as to avoid dangerous tensions due to the difference in expansion of the drum and wires. When the wires were reeled and put away after the measurement of the Kootenay base, the tem-

perature of the air was about 3° C. Consequently the drum was heated to about 20° to avoid subsequent excessive expansion.

The wires are also treated by another mechanical process, being a system of rythmic beating on the floor, which operation tends to produce a condition of stable equilibrium in the particles of the alloy and prevents unknown alterations in length from accidental disturbances such as might occur during long railway journeys and other rough usage.

Wire which are used under constant tension of say ten kg. are also tested by excessive tensions for periods of 24 hours, and any danger of accidental lengthening from sudden shocks or prolonged forces thereby removed.

*Standardizing Tests.*— When the wires have been treated by the operations described above, and mounted at their extremities by graduated scales they are then ready for comparison to determine their absolute lengths. The graduated invar scales are of a special design which is very ingenious.

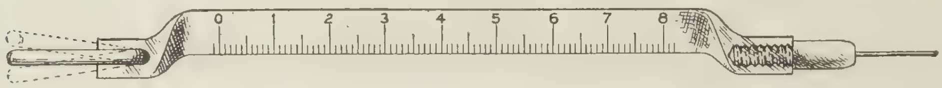


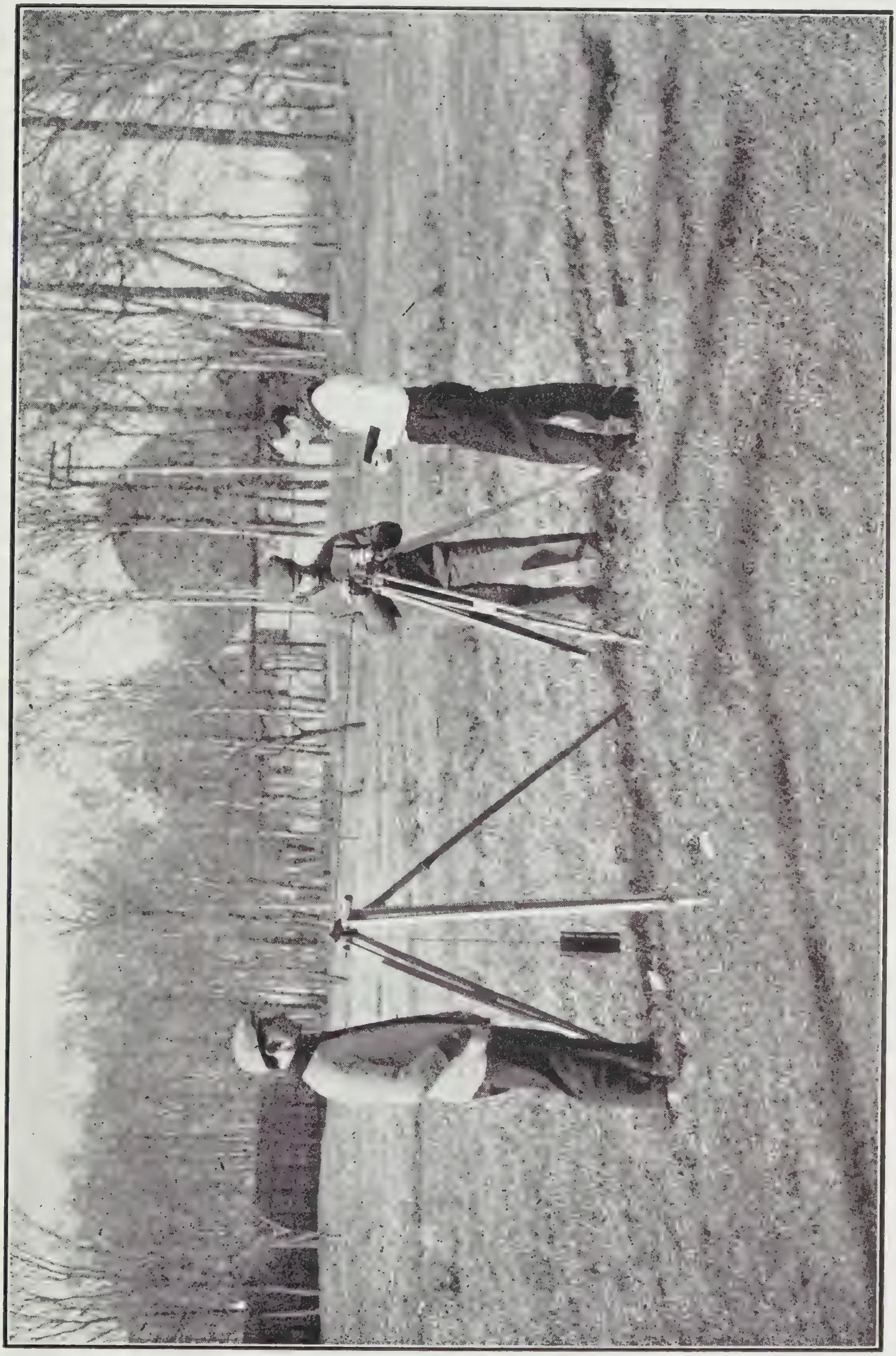

FIG. 3.

Figure 3 shows one of these scales, the graduated edge being hollowed so as to be in a direct line with the neutral axis of the wire. This form does away with errors in reading due to the terminal tangent to the curved wire not coinciding with the straight line joining the two extremities.

The standardization of the wires is made at the International Bureau, by exhaustive comparisons with the underground mural base, the wires being under conditions exactly similar to those under which they are used in field work.

The following certificate accompanied the wires received from the International Bureau of Weights and Measures by the Surveyor General of Dominion Lands.

PLATE II.



Taking Readings on the Invar Base Line Instrument

INTERNATIONAL BUREAU OF WEIGHTS AND MEASURES,

Pavillion de Breteuil, Sevres (S. and O.)

December 2, 1910.

CERTIFICATE

Of Invar Wires, Nos. 272 and 273.

Made by M. J. Carpentier, Paris. Belonging to the Surveyor-General, Canada.

*Description*

The wires, which are about 1·7 mm. in diameter, are terminated at each of their ends by invar scales, joined to the wires by a bolted piece to which they are screwed and rivetted.

Each of the scales is divided into mm. for a length of 82 mm., the centimetres being numbered from 0 to 8, the numbering being in the same direction at each end. The edge of the graduated scale is in a direct line with the neutral axis of the wire. One of the scales bears on its back this inscription : " J. Carpentier, Paris," and on the graduated side the numbers 272 and 273 respectively.

*Tests*

The wires were first submitted to a series of operations in order to assure the permanence of their lengths. Then they were compared to the mural base at the Bureau under the tension of 10 kg.

The series of comparisons were made from November 5, 1908, to January 4, 1909.

The wires were returned to the Bureau in September, 1910. They were immediately unrolled, and submitted to a new series of ten comparisons from September 19 to November 19, 1910. After which they were again rolled on their drum.

The comparisons were reduced to the mean temperature of 15° C. by means of the formula of dilatation

$$l_t = l_o \, ( 1 - 0{\cdot}000{,}000{,}121 \, t + 0{\cdot}000{,}000{,}000{,}15 \, t^2 )$$

determined from samples of these wires submitted to the same tension of 10 kg.

The following values were found for corresponding divisions on the scales :

Values of the Wires at 15° C. Under a Tension of 10 Kg.

| Date | Wire 272 | Wire 273 |
|---|---|---|
| January, 1909 | 24 m. + 1·10 mm. | 24 m. + 0·87 mm. |
| December, 1910 | 24 m. + 1.10 mm. | 24 m. + 0·94 mm. |

The determination of the dilatation of the wires was made by Mr. Ch. Ed. Guillaume, co-director of the International Bureau of Weights and Measures; the comparisons were made by Mr. Ch. Ed. Guillaume and Mr. L. Maudet, assistant at the Bureau.

(Signed) *J. Rene Benoit,*
Director of the Bureau.

The values of the wires for the measurements of Kootenay base was determined thus. The measurements were commenced on October 21, 1909. The two standardizations were made in December, 1908 and December 1910. The curve in figure 1, shows for October 21, 1909, an increase of 0·04$^{mm.}$ over the length in December, 1908. For wire 273, the second standardization in December, 1910, agreed almost perfectly with the increment corresponding to this date on the curve. Consequently, adding 0·04$^{mm.}$ to its value in December, 1908, as given by the certificate, we have, wire 273 = 24000·91$^{mm.}$ on October 21, 1909.

Wire 272, as will be observed in the certificate, did not lengthen according to the accepted law of the curve in figure 1. Indeed this wire showed in the second tests a length precisely the same as in the earlier ones. M. Guillaume thinks that the small permanent lengthening due to time was merely counteracted by a slight shortening induced by vibratory molecular motion during the long journey from France to British Columbia and return. It must be emphasized here, however, that we are discussing a very minute quantity, for in the measurements of Kootenay base, the values obtained by wires 272 and 273 differed only by 5·6 $^{mm.}$ in 8·5 kilometres.

To the value of each wire, we must add 0·01$^{mm.}$ for the periodic temperature dilatation, as explained before.

Therefore we have for the measurement of Kootenay base:

Wire 272 = 24001·11$^{mm.}$
Wire 273 = 24000·92$^{mm.}$

at the temperature 15° C, under a tension of 10 kg, on October 21, 1909.

*Principle of Measuring.*—The general principle followed in measuring is to find the number of times that a standard measure of length may be laid off along the line to be determined, marking the consecutive lengths as accurately as possible. The principle adopted by the International Bureau for measuring with invar wires, is the converse method, or in other words, to lay down courses having a certain approximate length, then to

measure accurately the length of these courses. This latter method is obviously the more precise.

The apparatus consists of twenty-four metre invar wires, having graduated end scales, a series of moveable tripods, clinometer accessories for reading the slopes of successive courses, and thermometers for recording temperature, besides such auxiliary instruments as pickets, level, levelling rods, aligning telescopes, plumb-bobs, &c.

*Moveable Tripods.*—Figure 4 shows a moveable tripod, the wooden frame A, having in its top a circular opening through which passes a smaller cylindrical metallic tube, B, which is

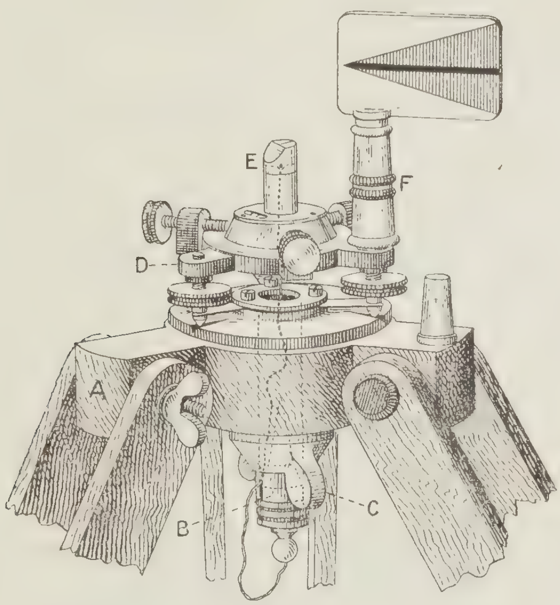



FIG. 4.

capable of a lateral displacement of several centimetres and may be fastened tightly to the wooden head by a thumb screw, C. The upper plate, D, bears three levelling screws held to the levelling plate by three springs. The datum mark, E, is also capable of a lateral motion by three horizontal screws, and carries a small level on its plate. The top of the datum mark is composed of a hard white alloy which will not oxidize. The cross forming the datum mark is of an ingenious design, having a bevelled edge, cut at the same angle as a cross-section of the end

scales, and permits the graduations to coincide with the horizontal plane of the datum mark. A plumb-bob, when not in use, screws into the tube, B, as in the figure, and the string passes through the hollow tube to a point directly under the cross of the datum mark. By means of this plumb-bob a definite point on the ground can be quickly and accurately transferred to the moveable datum mark. In the measurements of the Kootenay base, however, the transference was generally made by transit instrument.

The tripods, of which from six to eight are used, are set in position at approximately twenty-four metres apart, the alignment being made by small telescopes expressly intended for that purpose, or if hubs and tacks are set beforehand, the tripods can be fixed by means of the plumb-bobs.

A target, F, is placed on a vertical gudgeon beside the datum mark. On it readings of the slope are made with a small level hereinafter described. The flag is coated with white enamel, and in the apparatus devised by the International Bureau, the mark to be read upon was a horizontal black line six mm. wide. An improvement was made to these targets by painting a red triangle as in the figure. A better bisection can be made on the apex of the angle, and the red color assists greatly in rapid and precise reading.

*Slope.*— The difference in slope between successive tripods is measured by means of a small level shown in figure 5. The level replaces the flag on the upright gudgeon, the centre of the telescope being at the same height as the apex of the red target. At the focus of the object glass for a distance of twenty-four metres is a photographic scale etched on glass, the graduations being so spaced that one division covers twenty-four mm. at a distance of twenty-four metres, that is, reads a slope of one-tenth of one per cent. or ·001 in terms of the tangent. Readings are estimated to one-tenth of a division, or ·0001. Forward and backward readings are taken at each portée, or course, thus eliminating error of collimation.

On the Kootenay base, slopes up to five per cent. were read directly on the flags of the moveable tripods. Greater slopes than five per cent. were measured with the assistance of a long auxiliary rod, which when reading an ascending, or positive grade, was hung on the forward tripod with its flag close to the ground. By means of this rod the readings of the slope were reduced three per cent, and only the aplanatic part of the lens used. In reading negative or down grades, the auxiliary rod was placed above the low tripod, and the readings thus reduced five per cent. Wherever several slopes greater than five per cent were encountered, the heights of the end tripods above the hubs were measured with a tape, and by means of auxiliary levels with a transit the relative heights of the hubs were determined. The intermediate slopes read with the level were thus checked, and a close approximation obtained.

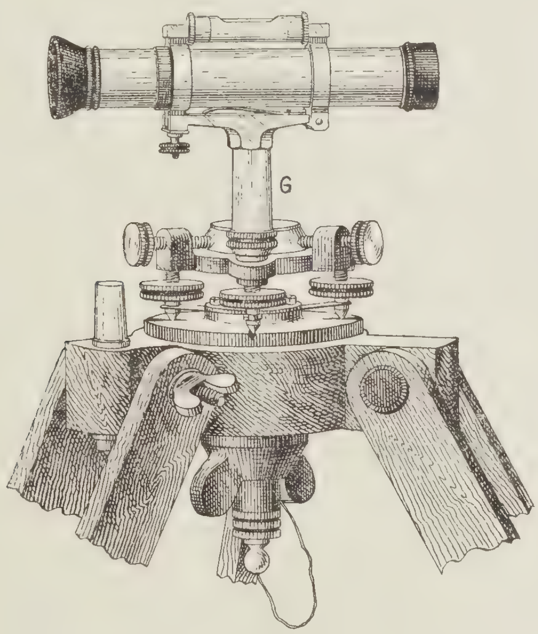



Fig. 5.

The level constants of the photographic scale were determined by sighting on a vertical rod placed at twenty-four metres. Numerous readings were taken under different weather conditions to allow for irregularity of light and refraction.

The correction for slope is made up of two parts. One is dependent on the horizontal projection of the straight line joining the tops of the tripods ; that is $l$ (1—cos. $\alpha$ ). This correction is given in table IV. in terms of tangent $\alpha$ for a slant length of twenty-four m. The table has been elaborately constructed up to ten per cent and may be quickly applied. In the last column of the table is also given the value of $l$ (1—cos. $\alpha$ ) for a length of one metre. When the distance between two successive tripods differs from twenty-four m. by an appreciable amount, such as from ten to fifty mm., the correction for slope as applied directly from table IV. must itself be modified by the small correction for the said excess or deficiency.

The second correction for slope (table V.) is due to the fact that when the ends of the wire are not at the same level the curve ceases to be symmetrical, or a true catenary. The correction for this deformation in the curve is a function of the slope and although very small, cannot be neglected as it is always in the same sense, that is, positive.

Tables IV. and V, are to be found in the appendix of 'La Mesure Rapide des Bases Géodésiques,' fourth edition.

In the first chapter of the same book may be found an exhaustive mathematical discussion on the theory of measuring with a stretched wire, with the alterations which take place in the catenary when the tension is changed by even a small amount, either by friction or by changes in terrestrial gravity.

The difference between the effects of terrestrial gravity at Sèvres, France, where the wires were standardized (latitude, 48° 50′ N.; elevation, 184 feet) and at the Kootenay base (latitude 51° 04′ N.; elevation, 2,700 feet) is so small that the resulting change in tension of the ten kilogramme weight dces not effect an appreciable alteration in the length of the wires.

*Tension Tripods.*—The wires are stretched for measuring by submitting them to a constant tension of ten kg., which is applied by means of weights suspended and hanging freely from window-sash cord passing over ball-bearing pulleys. The pulleys are supported by tension tripods as shown in figure 6. During

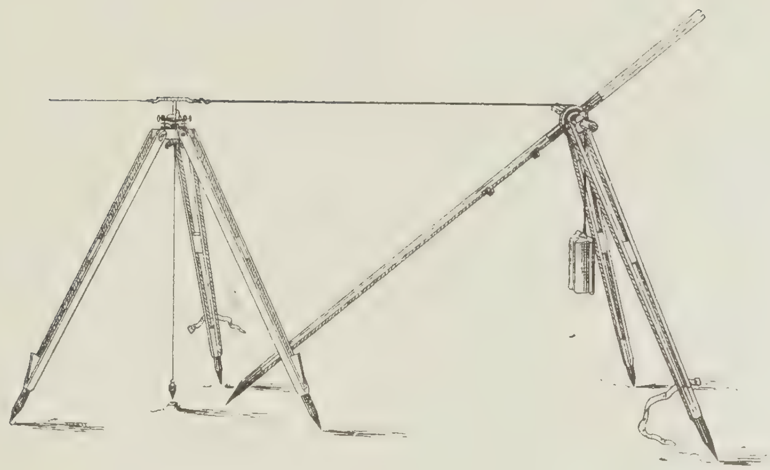


Fig. 6.

the measurements each tension tripod is placed in position by a man carefully drilled, and should be in the direct line of the base, and at such height that the graduated scale comes to within two or three millimetres of the datum mark. The ring at the end of the wire is attached to the tension cord by means of a hook and a snap and swivel.

*Readings on the Wire.*—An observer at each end of the wire reads the position of the datum mark on the graduated scale, pressing the latter with a gentle lateral motion against the datum mark. The scales are graduated in millimetres and readings are estimated to tenths. At least five readings are made at each end for every portée. The two scales are graduated in the same direction so that the reading at one end is always positive, and at the other is negative. The difference between the two readings gives the excess or deficiency from the true value of the wire. The readings in a set should rarely differ by more than 0·2 mm., a discrepancy greater than 0·3 mm. denoting an error in reading. In such a case the recorder orders extra readings to be made. Every so often the wires are reversed in direction end for end, or the observers interchange their positions, in order to eliminate personal equation, the observer at the positive end then making negative readings and *vice versa*. At each portée the readings are made on different portions of the scale, one

observer alternately shoving and pulling the wire, gently, but firmly, thus eliminating tensional friction in the pulleys or other accidental sources of error.

A tabulated form is used for all records and computations at each portée, a specimen page of which is here shown.

### Triangulation Base Measurement — Invar Apparatus — Records and Computations

| | |
|---|---|
| | Portée No. 278. |
| Recorder, Carson. | Date, Nov. 2, 1909. |
| Observers, { Rear (+), Carson. | Time, 10.30 A.M. |
| { Front (−), de la Condamine. | Direction, Southerly. |
| | Weather, Calm, Cloudy. |

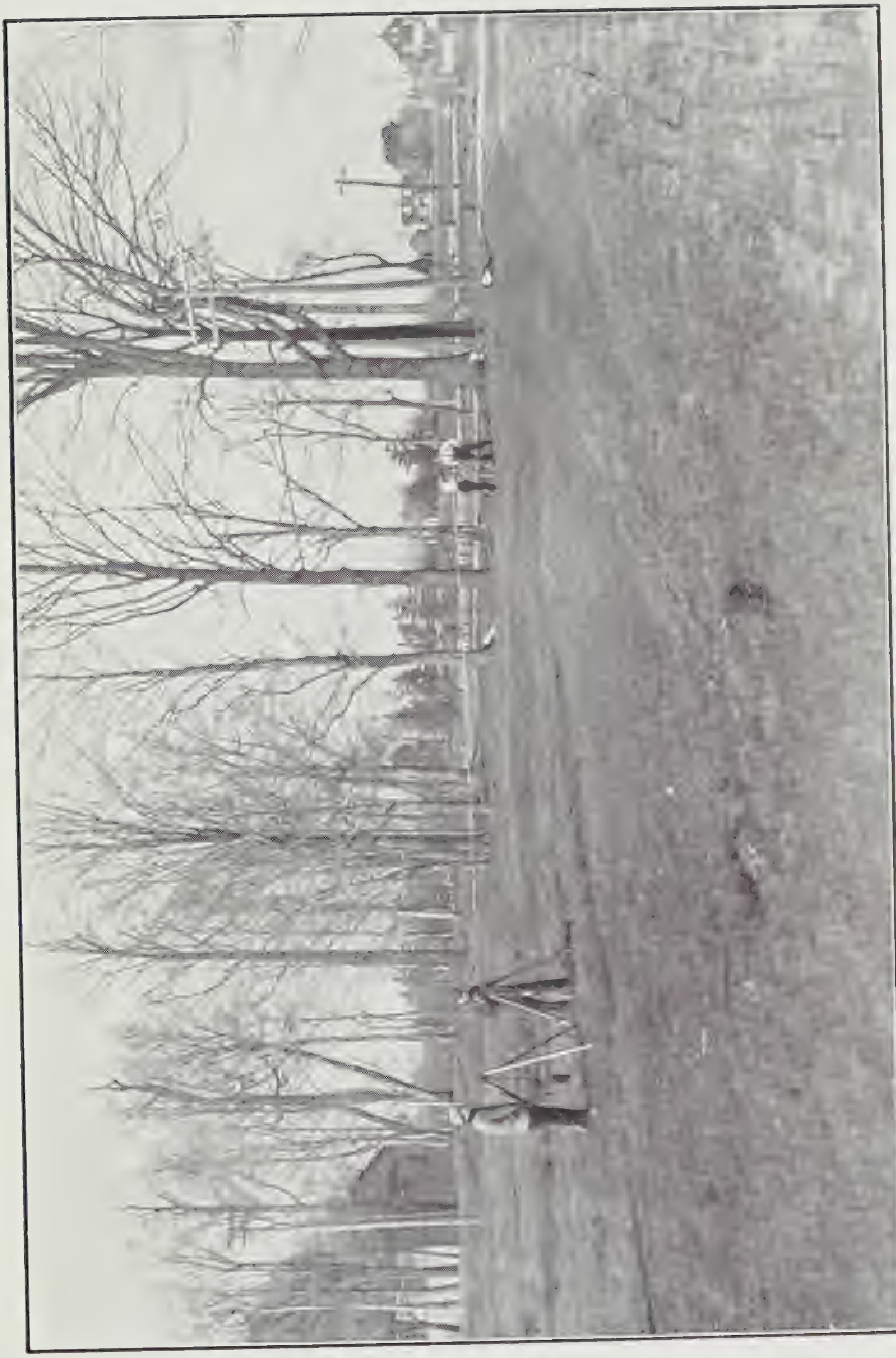
| No. | Readings: Wire No. 272 Rear (+) | Front (−) | R − F | Wire No. 273 Rear (+) | Front (−) | R − F |
|---|---|---|---|---|---|---|
| | | | mm. | | | mm. |
| 1 | 42·0 | 21·3 | +20·7 | 39·6 | 18·8 | + 20·8 |
| 2 | 59·0 | 38·4 | 20·6 | 46·4 | 25·6 | 20·8 |
| 3 | 36·7 | 16·0 | 20·7 | 62·8 | 41·8 | 21·0 |
| 4 | 50·9 | 30·2 | 20·7 | 46 0 | 25·1 | 20·9 |
| 5 | 32·8 | 12·1 | 20·7 | 58·3 | 37·4 | 20·9 |
| Mean. . . . . | | | +20·68 | Mean . . . | | +20·88 |

| Slope (Tangent): Forward | Back |
|---|---|
| + 0406 | − 0407 |
| 0406 | 0407 |
| Mean = | +04065 |
| Level Cor. = | 0003 |
| | +04035 |
| | mm. |
| Cor. Tab. IV | −19·465 |
| | − ·048 |
| | 19·513 |
| | − ·016 |
| | 19·529 |
| Cor. Tab. V + | ·005 |
| Total Corr. (−) | 19·524 |

| Temperature | °C. |
|---|---|
| | 7·2 |
| | 7·8 |
| Mean . . . | 7·5 |
| | mm. |
| Corr. (Table III) | +0·021 |

| | Wire No. 272 | Wire No. 273 |
|---|---|---|
| | mm. | mm. |
| Value of wire at 15° C. . . . . . . . . . . . . . . | 24001·11 | 24000·92 |
| Correction to 7°·5 . . . . . . . . . . . . . . . . . | + 0·021 | + 0·021 |
| Sum . . . . . . . . . . . . . . . . . | 24001·131 | 24000·941 |
| Readings . . . . . . . . . . . . . . . . . . . . . . . | + 20·68 | + 20·88 |
| Sum . . . . . . . . . . . . . . . . . | 24021·811 | 24021·821 |
| Correction for Slope . . . . . . . . . . . . . . . . | − 19·524 | − 19·524 |
| Value of Portée . . . . . . . . . . . . . . . . . . . | 24002·287 | 24002·297 |

PLATE III.



THE INVAR BASE LINE INSTRUMENT

## KOOTENAY BASE

Kootenay base controls the complete network of the triangulation survey in the railway belt, British Columbia, from the summit of the main range of the Rocky Mountains westward to the Cascade range.

The base lies in townships 19, ranges 19 and 20, west of the fifth meridian, on the right or easterly side of Columbia river, British Columbia, about twenty-one miles southeasterly from the town of Golden.

By means of three secondary stations the base is projected to the main triangulation, connecting with primary stations 17, 20 and 21, the simplicity and rigidity of the projection being almost ideal.

The approximate mean longitude of the base is 116° 39′ W; the mean latitude is 51° 04′ N approximately; the mean elevation is 823 metres above sea-level; the mean bearing is 309° 08′.

The length of the base, reduced to sea-level, is 8565·5735 metres.

Station A, marking the southerly end of the base, is 14·62 chains west, and 0·50 chains north of the wooden post marking the quarter section corner on the east boundary of section 16, township 24, range 19.

Station B, marking the northerly end of the base is 20·53 chains west and 12·28 chains south of the iron post and mound marking the northeast corner of section 35, township 24, range 20.

The end of the base, or geodetic point, at station A is the intersection of a pair of fine lines at right angles to each other, stamped in the head of a brass bolt six inches long and three-quarters of an inch in diameter with a flat head one and one-half inches square. This bolt is set in concrete three feet beneath the surface of the ground and is covered with loose earth. The head of the bolt also bears the letter A stamped upon it. There is no surface mark, except four iron reference bolts fifteen inches long and one inch in diameter which bear north, east, south and west respectively from the geodetic point, and are each distant three feet from it.

Station B is similarly marked except that the bolt bears the letter B stamped upon its head.

The base line skirts the westerly edge of the Beaverfoot range of mountains close to the bottom lands of Columbia valley. It crosses the Government wagon road no less than ten times, and also intersects the surveyed line of the projected Kootenay Central railway at several points. The base runs mostly through uncleared land covered with second-growth poplar and birch, crossing also several cultivated fields, and through occasional patches of spruce and fir averaging eighteen inches. The line was cleared of all timber, grass and brush for a width of six feet, large stumps being sawn off level with the ground. Beginning

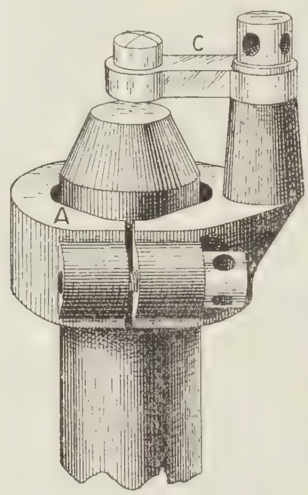



FIG. 7.

at the northerly end of the base, hubs twenty-four metres apart were set with a standard chain supplied by the makers of the base apparatus. This chain which is composed of twisted wire strands, was found to be unsatisfactory, as its length varied considerably with the temperature. The chain also soon became twisted and kinked with repeated windings and unwindings on the reel; and when the measurements were afterwards made with the invar wires the distance between some of the hubs differed from twenty-four metres by as much as fifty mm. Tacks

were placed in the hubs, the alignment being made with an ordinary four-inch transit. . A forward line was run, sighting being made on the signal at station A, hence the errors of alignment were very small. The hubs were set along the whole length of the line and the southerly end of the base as previously established was changed and set at an integral number of twenty-four metre stretches. This change was necessary as only twenty-four metre wires were available for measuring, although with auxiliary wires of say four and eight metres any short distance can be conveniently determined.

The base was divided into six sections by means of controlling datum marks supplied with the apparatus. A cut of one of these datum marks is shown in figure 7. It consists of a round iron peg, pointed at one end, about eighteen inches long and one and one-half inches in diameter, which may be driven firmly into the ground. The collar A, which is fastened near the top of the peg by capstan screws, has a vertical gudgeon B, which again bears a revolving horizontal arm C. The end of this arm carries a small cross which serves as a datum mark. By the double motion of the two sets of screws this cross can be set very accurately on line, and within several millimetres of the desired longitudinal distance. It is not necessary to fix the datum mark accurately for distance, as, once it is in place, the distance to and from it is measured by setting a tripod accurately over it.

By dividing the base line into sections in this manner, six closing checks on the measurements were obtained, each section being considered separately. In determining the probable error of the adopted length of the whole base the probable errors of each section line were combined. The base was divided according to the configuration of the ground, each section being about the length covered in a day's work.

The base was measured in a northerly direction with a single wire, Nos. 272 and 273 being used at alternate sections. A complete double measurement was also made in a southerly direction using both wires. The method of using two wires is somewhat slower,

but serves to check the relative lengths of the wires, mistakes in reading and other sources of error.

The weather during the measurements was exceptionally suitable for this work. The days were mostly cloudy, with rain at night. The range of temperatures on some days was only about five degrees, 3° C to 8° C, with both rising and falling temperatures. On three days the weather was clear and the temperature rose from 3° C to 20° C, falling again to 7° C in the afternoon. Calm weather prevailed with occasional light winds, but not of sufficient freshness to disturb the wires. On two days rain fell during the measuring, but it is not thought that the sag of the wires was appreciably increased by the weight of the clinging rain-drops.

The distribution of the staff of seven engaged in the measurements was as follows : one man setting tripods over the hubs, one porter carrying tripods from rear to front, one man at each of the two tension tripods, two observers, one of whom acted also as recorder and leveller, and one porter carrying an end of the wires in lieu of the observer who performed the levelling. The work would have been materially expedited had a competent man been available to take the levels, this duty falling on one of the observers whose manifold duties retarded progress. Before commencing the measuring, the party was drilled with a practice wire until all became thoroughly familiar with the work, and acted together as a team with preconcerted signals. Before the completion of the base a speed of four hundred metres per hour was attained, when using one wire, which is indeed satisfactory, considering the nature of the ground on a base line in the Rocky Mountains.

*Results of Measurements.*— The results of the measurements of the Kootenay base have been tabulated in the accompanying table.

The second standardization of wires 272 and 273 showed that the latter wire had responded almost perfectly to the curve in figure 1, and M. Guillaume suggested that the measurements

with wire 273 should be taken as correct and the measurements with wire 272 as a check. Consequently the observed measurement obtained with wire 273, viz.: 8,566,677·829 millimetres has been adopted for Kootenay base. The error of this value due to

TABULATION OF MEASUREMENTS OF KOOTENAY BASE

| Section | Portees | No. of Portees | Direction of Measurement | Date 1909 | Temperature Range | Mean Temp. | Weather | Wire No. | Length of Section | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | ° ° ° | ° | | | mm. | mm. |
| I | 1-60 | 60 | N | Oct. 21 | 3 to 20 to 7 | 12 | Calm, cl'dy to fair | 272 | 1,441,007·529 | 1·832 |
| | | | S | Oct. 22 | 7 to 14 to 5 | 10 | Calm, cloudy, with | 272 | 1,441,005.697 | |
| | | | S | Oct. 22 | " " " | | occasional breeze | 273 | 1,441,007·307 | |
| II | 61-109 | 49 | N | Oct. 23 | 3·4 to 7·5 | 6 | Calm, cloudy, rain | 272 | 1,176,324·003 | 0·875 |
| | | | S | Oct. 25 | 2 to 13 to 7 | 9 | Calm, cl'dy to fair | 272 | 1,176,323·128 | |
| | | | S | Oct. 25 | " " " | | " " " " | 273 | 1,176,323·188 | |
| III | 110-181 | 72 | N | Oct. 26, 27 | 3 to 20 to 14 | 12 | Calm, cl'dy, light | 273 | 1,728,535·610 | 2·548 |
| | | | S | Oct. 26 | 6 to 12 to 0 | 7·5 | Calm, cl'dy [wind | 272 | 1,728,537·238 | |
| | | | S | and 28 | " " " | | Calm, foggy | 273 | 1,728,538·158 | |
| IV | 182-223 | 42 | N | Oct. 29 | 9 to 20 to 9 | 13 | Calm, cl'dy to fair | 273 | 1,007,437·097 | 1·129 |
| | | | S | Oct. 30 | 6 to 11 to 3 | 7·4 | Calm, cloudy, oc- | 272 | 1,007,437·456 | |
| | | | S | " " | " " " | | casional breeze | 273 | 1,007,438·226 | |
| V | 224-282 | 59 | N | Nov. 1 | 3 to 7 to 4 | 6 | C'lm, cl'dy, oc. br. | 272 | 1,414,993.029 | 0·865 |
| | | | S | Nov. 2 | 6 to 8 to 6 | 7 | Calm, cl'dy, some | 272 | 1,414,993·894 | |
| | | | S | " " | " " " | | showers | 273 | 1,414,994·824 | |
| VI | 283-357 | 75 | N | Nov. 3 | 5 to 10 to 5 | 7 | Calm, cloudy | 273 | 1,798,377·308 | 1·182 |
| | | | S | Nov. 4 | 5·5 to 15 to 2 | 7 | Calm, cl'dy to fair | 272 | 1,798,374·814 | |
| | | | S | and 6 | " " " | | Calm, fair | 273 | 1,798,376·126 | |

Total number of portées = 357

| | mm. |
|---|---|
| Length of Base (with wires 272 and 273 used at alternate sections) = | 8,566,674·576 |
| " " " with wire No. 272 . . . . . . . . . . . . . . . . . . . . . | 8,566,672·227 |
| " " " with wire No. 273 . . . . . . . . . . . . . . . . . . . . . | 8,566,677.829 |

At M. Guillaume's suggestion greater weight has been given to wire No. 273, and the value of the base as determined with this wire has been adopted as the true value of the base = 8,566,677·829 mm.

Correction to ocean level = − 1104·3

Final value of base (in mm.) = 8,565,573·5

Probable error of absolute length of wire 273 = 0

Probable error due to measurements =

$$\frac{1}{2}\sqrt{\frac{357}{6}\left(\frac{1{\cdot}832^2}{60}+\frac{0{\cdot}875^2}{49}+\frac{2{\cdot}548^2}{72}+\frac{1{\cdot}129^2}{42}+\frac{0{\cdot}865^2}{59}+\frac{1{\cdot}182^2}{75}\right)} = \pm\ 1{\cdot}8 \text{ mm.}$$

comparisons is inappreciable. The error due to measurements (*i.e.* slope, temperature, readings on the wires, setting of tripods, etc.) has been determined as follows: The difference between the two lengths of each section of the base obtained by the same wire is due to measurements only. These differences for each section have been combined to obtain the probable error of the whole base due to the measurements, each section being given a weight corresponding to the number of portées it contains.

The length of the base has been reduced to sea-level, the correction being a function of the measured length, the elevation above the sea, the latitude, and the bearing. The correction for this base is 1·1043 metres.